Section 7.9: Impulses and the Dirac Delta Function

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Informal things to know about the Dirac Delta Function

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Informal things to know about the SHIFTED Dirac Delta Function

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Informal things to know about the (shifted) Dirac Delta Function

Ex 0: Calculate the following integrals…

a) $\int_{-\infty}^{\infty} \cos t \ \delta(t) \ dt$

b) $\int_{2}^{10} \ln t \ \delta\left(t - e^2\right) dt$

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Informal things to know about the (shifted) Dirac Delta Function

Ex 0.5: Find the Laplace Transform of the generalized function $t^2\delta(t-3) - (\cos t)\,\delta(t)$

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Informal things to know about the (shifted) Dirac Delta Function

Example 1 A mass attached to a spring is released from rest 1 m below the equilibrium position for the mass–spring system and begins to vibrate. After π seconds, the mass is struck by a hammer exerting an impulse on the mass. The system is governed by the symbolic initial value problem

$$\text{(11)} \qquad \frac{d^2x}{dt^2} + 9x = 3\delta(t-\pi)\,; \qquad x(0) = 1\,, \qquad \frac{dx}{dt}(0) = 0\,,$$

where $x(t)$ denotes the displacement from equilibrium at time t. Determine $x(t)$.

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A little more of the story…

Physics: IMPULSE (constant force)

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A little more of the story…

Physics: IMPULSE (variable force)

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A little more of the story…

Dirac Delta function is “like” a limit of a sequence of functions

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A little more of the story…

What does it mean to solve a DE with a generalized function?

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A little more of the story…

Why is $\int_{-\infty}^{\infty} f(t)\delta(t)\,dt = f(0)$?